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The quenched generating functional for hadronic weak interactions

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The ultraviolet behaviour of the generating functional for hadronic weak interactions with $|\Delta S| = 1, 2$ is investigated to one loop for a generic number of flavours and in the quenched approximation. New quenched chiral logarithms generated by the weak interactions can be accounted for via a redefinition of the weak mass term in the $\Delta S = \pm 1$ weak effective Lagrangian at leading order. Finally, we illustrate how chiral logarithms are modified by the quenched approximation in $K \rightarrow \pi\pi$ matrix elements with $\Delta I = 1/2$ and $3/2$.

1. Introduction

The computation of weak matrix elements at long distances is one of the main and still open problems in lattice QCD. Much progress has been done, although their computation still suffers from the presence of major sources of systematic errors: finite volume effects, unphysical quark masses, the need of computing *unphysical* matrix elements [1] (see also [2] for recent alternative proposals) and finally the fact that still most of the lattice evaluations are done in the quenched approximation. Here we report on some results concerning formal aspects of the quenched approximation in the ultraviolet regime. The framework we use is known as Chiral Perturbation Theory (ChPT) [3] and quenched ChPT [4,5], extended to full (unquenched) weak interactions in [6] and to the quenched case in [7]. The advantages of deriving the generating functional of the effective low energy theory in the full and quenched case are at least two: first, it allows for a systematic control on the quenched modifications and second, it gives in one step the coefficients of all chiral logarithms for any Green's function or S-matrix element.

2. The weak generating functional to one loop

The quenched generalization of the weak effective Lagrangian for nonleptonic weak interactions

with $|\Delta S| = 1, 2$ and at leading order in the chiral expansion can be written as follows

$$\begin{aligned} \mathcal{L}_{\Delta S=1} = & \tilde{V}_8(\Phi_0) \text{str}(\Delta_{s32} u_{\mu s} u_s^\mu) \\ & + \tilde{V}_5(\Phi_0) \text{str}(\Delta_{s32} \chi_{+s}) \\ & + \tilde{V}_0(\Phi_0) \text{str}(\Delta_{s32} u_{\mu s}) \text{str}(u_s^\mu) + \text{h.c.} \\ & + \tilde{V}_{27}(\Phi_0) t^{ij,kl} \text{str}(\Delta_{sij} u_{\mu s}) \text{str}(\Delta_{skl} u_s^\mu) \end{aligned} \quad (1)$$

and the same last term for $\Delta S = 2$ with the appropriate $t^{ij,kl}$ tensor. The projection matrix onto the octet and 27-plet components is the graded matrix $\Delta_{sij} = u_s \lambda_{ij} \frac{1+\tau_3}{2} u_s^\dagger$, with $(\lambda_{ij})_{ab} = \delta_{ia} \delta_{jb}$. The graded fields $u_{\mu s} = i u_s^\dagger D_\mu U_s u_s^\dagger = u_{\mu s}^\dagger$ and $\chi_{+s} = u_s^\dagger \chi_s u_s^\dagger + u_s \chi_s^\dagger u_s$ contain the dynamical meson field $U_s \equiv u_s^2 = \exp(\sqrt{2}i \Phi/F)$ in the usual way. The field $\chi_s = 2B_0 \mathcal{M}_s + \dots$ contains the graded quark mass matrix. The potentials $\tilde{V}_i(\Phi_0)$ are real and even functions of the super- η' field $\Phi_0 = \text{str}(\Phi)$ and we keep them up to order Φ_0^2 . The ultraviolet divergences of the weak generating functional to one loop can be derived as it was done in the strong sector [5], provided an expansion in powers of G_F is performed. For degenerate quark masses we get for the $\Delta S = \pm 1$ octet contributions at order G_F :

$$\begin{aligned} Z_{(8)}^q = & -\frac{1}{(4\pi)^2(d-4)} \int dx \left\{ \frac{1}{6} m_0^2 (g'_8 - 2g_8) k \langle \Delta \chi_+ \rangle \right. \\ & + \frac{1}{4} g_8 W_4 + \frac{1}{8} W_6 + \left(\frac{\alpha}{12} (g_8 - g'_8) - \frac{1}{8} \bar{g}_8 \right) (W_{12} + W_{36}) \\ & \left. + \left[-\frac{3}{16} g_8 \left(1 - \frac{16}{3} v_1 \right) + \frac{1}{8} g'_8 (1 - 4v_1) \right] W_7 \right\} \end{aligned}$$

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$$\begin{aligned}
& + \frac{1}{16}(g_8 - 8\tilde{v}_8)W_8 + \left(\frac{\alpha}{12}(2g_8 - g'_8) - \frac{1}{8}\bar{g}_8 \right) W_{10} \\
& + \left[\left(\frac{1}{8} + \frac{\alpha^2}{36} \right) (g'_8 - g_8) - \frac{1}{2}v_2(g'_8 - 2g_8) - \frac{1}{2}\tilde{v}_5 \right. \\
& \left. + \frac{\alpha}{12}\bar{g}_8 \right] W_{11} + \left(-\frac{\alpha}{6}g_8 + \frac{1}{4}\bar{g}_8 \right) (W_{21} + W_{22}) \}.
\end{aligned}$$

The operators W_i are listed in Appendix A of [7]. The 27-plet contribution is given by

$$\begin{aligned}
Z_{(27)}^q = & -\frac{1}{(4\pi)^2(d-4)} \int dx g_{27} \left\{ \frac{1}{12}D_1 + \frac{5}{6}D_2 \right. \\
& - \frac{1}{8}D_3 - \frac{7}{24}D_4 - \frac{1}{4}D_5 + \frac{1}{4}D_6 + \frac{1}{2}D_7 + \frac{3}{8}D_8 \\
& + \frac{1}{4} \left(1 - 2\frac{\tilde{v}_{27}}{g_{27}} \right) D_{10} - \frac{3}{8}D_9 - \frac{1}{8} \left(1 - \frac{2}{3}\alpha \right) D_{12} \\
& - \frac{1}{4}D_{11} - \frac{1}{12}D_{13} + \frac{1}{12}D_{14} - \frac{1}{24}D_{15} - \frac{1}{12}D_{16} \\
& + \frac{7}{12}D_{17} + \frac{7}{12}D_{18} - \frac{1}{4}D_{19} - \frac{1}{4}D_{20} + \frac{1}{6}D_{21} \\
& \left. + \frac{1}{6}D_{22} + \frac{1}{4}D_{23} + \frac{1}{4}D_{24} \right\} + O(G_F^2),
\end{aligned}$$

where the operators D_i are listed in Appendix A of [7]. m_0^2 and α are the usual singlet parameters. The couplings g_8, g'_8, \bar{g}_8 and g_{27} are the first terms in the expansion of the weak potentials \tilde{V}_i , $i = 8, 5, 0, 27$, while the parameters \tilde{v}_i are the coefficients of the Φ_0^2 term. The quenched generating functional for $\Delta S = 2$ interactions has exactly the same structure of $Z_{(27)}^q$, with the appropriate $t^{ij,kl}$ tensor.

The quenched approximation largely reduces the ultraviolet divergent contribution to the octet sector at one loop. Of the initially divergent 25 octet operators only 10 remain in the quenched approximation. The octet operators W_5, \dots, W_{12} and W_{38} contribute to $K \rightarrow 2\pi$ decays. In the 27-plet sector all the unquenched chiral invariants D_i survive to the quenched approximation. Only the operators D_8, \dots, D_{12} contribute to $K \rightarrow 2\pi$ decays.

The systematic cancellation of the flavour number dependence follows the rules outlined in [5,8]. The cancellation of the $1/N, 1/N^2$ terms is provided by the sum of the non-singlet contributions and the singlet contributions within the bosonic sector. This type of cancellation is entirely due

to the presence of a dynamical singlet field. The linear flavour number dependence is cancelled by the fermionic ghost determinant as expected.

2.1. Quenched chiral logarithms

The most relevant behaviour of the quenched generating functional to one loop is the appearance of quenched chiral logarithms, i.e. of the type $m_0^2 \log m_\pi^2$, that are pure artefacts of the quenched approximation. As it was discussed at length in [5], quenched chiral logarithms appearing in the strong sector can be formally reabsorbed in a redefinition of the B_0 parameter. In the weak sector an analogous mechanism occurs. The quenched chiral logarithms which appear through the first term in the equation for $Z_{(8)}^q$ can be formally reabsorbed into a redefinition of the weak mass term coupling g'_8 of the leading order Lagrangian (1). To remove the m_0^2 divergence one has to add to the lowest order parameter g'_8 a d -dependent part proportional to m_0^2 that has a pole at $d=4$ and the rescaled coupling can be defined as follows:

$$\begin{aligned}
g'_{8R} = & \\
g'_8 \left(1 - \frac{m_0^2}{48\pi^2 F^2} \left(1 - 2\frac{g_8}{g'_8} \right) \log \frac{M^2}{\mu^2} + \delta g'_8(\mu) \right).
\end{aligned}$$

The rescaling of the coupling g'_8 together with the rescaling of the parameter $B_0 \rightarrow \bar{B}_0$ defined in [5] in the tree level contribution to any weak observable can be used as a short-cut procedure to unveil the presence of quenched chiral logarithms, generated when the quenched approximation is implemented to one loop.

3. $K \rightarrow \pi\pi$ matrix elements

Using the one loop expression of $K \rightarrow \pi\pi$ matrix elements as derived in [9] and the quenched counterpart of the ultraviolet divergences derived here, we can produce a few quantitative estimates of quenching effects on the coefficients of chiral logarithms that contribute to $K \rightarrow \pi\pi$ amplitudes. We work in the infinite volume limit for illustrative purpose. In [10] we shall report on the analysis of *unphysical* choices of the kinematics used for the computation of lattice $K \rightarrow \pi\pi$ matrix elements.

Table 1

The coefficients of the one loop chiral logarithms in $K \rightarrow \pi\pi$ amplitudes, full and quenched.

	Full ($m_K = m_\pi$)	Full ($m_\pi = 0$)	Quenched
A_0^8	$\frac{11}{18}$	$-\frac{5}{4}$	$-(\frac{1}{2} + \frac{4}{9}\alpha^2 - 2\alpha) + v_i, \tilde{v}_i, \bar{g}_8, g'_8$
A_0^{27}	$-\frac{17}{4}$	$-\frac{15}{2}$	$-4(1 - \frac{\alpha}{2})$
A_2	$-\frac{11}{2}$	$-\frac{3}{4}$	$-\frac{13}{4}$

We decompose the $K \rightarrow \pi\pi$ matrix elements into definite isospin invariant amplitudes: $\Im m A_0 \equiv \Im m(A_0^8 + A_0^{27})$ with $\Delta I = 1/2$, and $\Im m A_2$ with $\Delta I = 3/2$. In Table (1) we compare the coefficients of chiral logarithms for the three amplitudes in the full theory and in the quenched approximation. For the full amplitudes we consider two extreme mass configurations in the one loop corrections: a) degenerate masses, i.e. $m_K = m_\pi$ and b) $m_\pi = 0$. The numerical analysis of the physical non degenerate mass case together with the comparison with *unphysical* choices of the matrix elements on the lattice will be given in [10]. The coefficients of chiral logarithms in the full 27-plet amplitudes $\Im m A_0^{27}$ and $\Im m A_2$ are quite large in the degenerate mass limit. In addition, by the comparison of second and third column in Table (1) one can conclude that all the coefficients in the full amplitudes are extremely sensitive to the variation of masses. Using $\alpha \simeq 0.6$ and disregarding for now the unknown contributions to the octet amplitude in the quenched case we find the following pattern going from the full degenerate mass case to the quenched one. Quenching reduces by a tiny amount (from 0.6 to 0.54) the coefficient of the chiral logarithm in the octet amplitude A_0^8 , while reduces the one in A_{27} by about 34% in absolute value and 41% in the case of A_2 . Note that the pattern is opposite for A_2 when we compare the quenched amplitude to the $m_\pi = 0$ limit of the full amplitude. This analysis (see [7,10] for more details) shows that quenching largely affects the coefficient of the chiral logarithm in the 27-plet $\Delta I = 3/2$ and $\Delta I = 1/2$ amplitudes. In addition, the comparison with the $m_\pi = 0$ limit of the full

amplitudes (expected to be the most approximate to the physical value) shows that the modification induced by quenching follows a pattern that tends to suppress the $\Delta I = 1/2$ dominance.

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